Teacher notes
Topic E

The mass-luminosity relation

In B6 we showed that an approximate expression for the pressure (assumed uniform) inside a star is

$$P = \frac{3}{8\pi}\frac{GM^2}{R^4}$$

Inserting into the ideal gas law we find (*m* is the mass an atom of the material making up the star)

$$PV = Nk_B T$$

$$T = \frac{PV}{Nk_B} = \frac{\frac{3}{8\pi}\frac{GM^2}{R^4}V}{\frac{M}{m}k_B} = \frac{\frac{3}{8\pi}\times\frac{GM^2}{R^4}\times\frac{4\pi R^3}{3}}{\frac{M}{m}k_B} = \frac{1}{2}\frac{GMm}{Rk_B}$$

Now, the luminosity is given by the Stefan-Boltzmann law:

$$L = \sigma 4\pi R^2 T^4 = \sigma 4\pi R^2 (\frac{1}{2}\frac{GMm}{Rk_B})^4 = \frac{\sigma\pi G^4 m^4}{4k_B^4}\frac{M^4}{R^2}$$

Since $\rho = \frac{3M}{4\pi R^3} \Rightarrow R = (\frac{3M}{4\pi\rho})^{\frac{1}{3}}$ and so

$$L = \frac{\sigma\pi G^4 m^4}{4k_B^4}\frac{M^4}{(\frac{3M}{4\pi\rho})^{\frac{2}{3}}} \propto M^{3.3}$$

This shows that for stars of constant density the luminosity is proportional to the 3.3 power of the mass.

The approach above is very crude and only approximately correct.

It does however explain why heavy stars spend less time on the main sequence than lighter stars: roughly, $L = \frac{E}{T}$, where *E* is the total energy radiated by the star and *T* its lifetime on the main sequence. The energy comes from fusion reactions and we expect $E \approx Mc^2$. Then

$$M^{3.3} \approx \frac{Mc^2}{T} \Rightarrow T \approx \frac{1}{M^{2.3}}$$

i.e. the larger the mass, the smaller the time on the main sequence.